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UV asymptotically free QED as a broken YM theory in the unitary gauge

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Abstract

We compute the β-function of a YM theory, broken to $U(1)$, by evaluating the coupling constant renormalization in the broken phase. We perform the calculation in the unitary gauge where only physical particles appear and the theory looks like a version of QED containing massive charged spin 1 particles. We consider an on-shell scattering process and after verifying that the non-renormalizable divergences which appear in the Green's functions cancel in the expression of the amplitude, we show that the coupling constant renormalization is entirely due to the photon self-energy as in QED. However we get the expected asymptotic freedom and the physical charge decreases logarithmically as a function of the symmetry breaking scale.

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Here we present a computation of the renormalization group β-function for a QED-like theory, i.e. an unbroken $U(1)$ gauge theory, in which there are charged massive vector bosons. Renormalizability requires it to be the broken phase of a non-abelian theory (which here for definiteness we consider to be an $SU(2)$ gauge theory with a Higgs in the adjoint representation, i.e. the well known Georgi-Glashow model [1]). Since the β function is related to the high energy properties of the model, it is expected to coincide with the asymptotically free one computed in the symmetric phase [2]. Here we perform the calculation from the point of view of the low energy $U(1)$ phase, choosing the unitary gauge and thus including in the loops only physical and (except for the $U(1)$ photon) massive particles. As far as we know, this computation has not been done before. From one side, it allows to express the β function in terms of the contributions of the physical degrees of freedom of the broken phase while, from the other side, it illustrates the subtle mechanism by which an abelian theory (the unbroken $U(1)$) with massive charged spin-1 bosons manages to be asymptotically free. Recently, this property has been shown to play a key role in understanding the duality properties of the low energy phase of the $N = 2$ SY theory (see [3]). There, what is relevant is the dependence of the effective coupling constant on the expectation value of the Higgs field, proportional to the charged spin-1 particle mass. We will see that this dependence comes out rather directly in our computation. It is usually stated that the unitary gauge represents an unconvenient choice for performing perturbative calculations because of the bad high energy behavior of the massive gauge particle propagator which causes the occurrence of non renormalizable divergences in the computation of Green's functions. We will show here that this fact does not represent any technical difficulty, but on the contrary, the computation of the β function turns out to be remarkably simple. Actually, our understanding of this point has greatly benefitted from a very recent paper by J. Papavassiliou and A. Sirlin [4],(see also [5]) which indicates what to expect for the cancellation mechanism of non-renormalizable divergences that is known to hold for the computation of gauge invariant on-shell quantities [6]. We make use of this point of view by defining the physical coupling constant as the residue of the pole at zero momentum transfer of the on-shell scattering amplitude for two charged fermions. After verifying the expected cancellation of non renormalizable divergences (for

which we make use of available results for the computation of the various graphs involved [8]), we show that the β function can be extracted just from the photon self-energy. In particular, its asymptotically free part comes from a single graph, namely the photon self-energy due to the charged massive vector boson. If we compare the result with the standard β function computation in the unbroken phase, we see that this graph alone reproduces the sum of the contributions to the β-function of the vector bosons and the Higgs scalars.

It is perhaps worthwhile to notice that the same type of procedure, namely considering scattering amplitudes to extract the β-function, is also the starting point of string theory computations, see [7], since there, in principle, only on-shell S-matrix elements are defined.

To be specific we consider a $SU(2)$ theory broken to $U(1)$ by a Higgs in the adjoint representation and containing two families of Dirac fermions in the fundamental representation. The corresponding Lagrangian is:

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{\lambda}{4}(\Phi^a\Phi^a - v^2)^2 + \\ + \overline{\psi}_f(i\not{D} - m_f)\psi_f \ , \tag{1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc}A^b_\mu A^c_\nu \ , \tag{2}$$

$$D_\mu\Phi^a = \partial_\mu\Phi^a - g\epsilon^{abc}A^b_\mu\Phi^c \ , \tag{3}$$

$$D_\mu\psi_f = \left(\partial_\mu + i\frac{g}{2}\tau^a A^a_\mu\right)\psi_f \ , \tag{4}$$

with τ^a the Pauli matrices, ($a = 1, 2, 3$) and $f = 1, 2$ a family index. After the breaking $SU(2) \Longrightarrow U(1)$, we have in the unitary gauge:

$$\Phi^1 = \Phi^2 = 0 \ , \qquad \Phi^3(x) = v + \phi(x) \ . \tag{5}$$

Defining $W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)$ to be the charged vector boson fields and $A_\mu = A^3_\mu$ to be the photon field, we can rewrite the Lagrangian in the form of a $U(1)$ theory:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu|^2 - M_W^2 W^+_\mu W^{-\mu} + \overline{\psi}_f(i\not\partial - m_f)\psi_f +$$

$$+ig(\partial^\mu A^\nu - \partial^\nu A^\mu)W^+_\nu W^-_\mu + (igA^\mu(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^{-\nu} + h.c) + g^2(A^\mu A^\nu W^+_\mu W^-_\nu - A_\nu A^\nu W^+_\mu W^{-\mu}) +$$

$$-\frac{g}{2}A_\mu \overline{\psi}_f \gamma^\mu \tau^3 \psi_f - \frac{g}{\sqrt{2}}(W^+_\mu \overline{\psi}_f \gamma^\mu \tau_+ \psi_f + h.c.) + \cdots \tag{6}$$

Here $\tau_+ = 1/2(\tau^1 + i\tau^2)$ and $M_W = gv$ is the mass acquired by the W bosons after symmetry breaking. In eq.6, we have omitted the terms of the lagrangian which do not contribute, at one loop, to the process we are interested in. Clearly, in the unitary gauge all the fields appearing in the Lagrangian correspond to physical particles. One can think of the Lagrangian, eq.6, as describing a particular version of QED including a massive charged vector boson.

As we said above, we will consider a definite physical process, the on-shell elastic scattering of a positively charged fermion of the first family off a negatively charged fermion of the second family. Let us notice that the scalar field is not coupled to the fermions and that it is not involved in any of the diagrams that contribute to this process at the 1-loop order. In addition, as we consider the elastic scattering of two distinguishable fermions, there are neither annihilation nor exchange channels.

Figure 1 displays the graphs involving W bosons which contribute to the process at the order g^4 (Of course, to get the full amplitude, one has to add also the ordinary diagrams of spinorial electrodynamics, which are not shown for brevity). These graphs can be grouped in four classes:

1. the graphs P_1, P_2 represent the W contribution to the photon self-energy. P_2 is a tadpole like graph and only P_1, where a $W^\pm$ pair is created and then annihilated, gives rise to the photon wave function renormalization.

2. the graphs $E_1, \cdots, E_4$ represent the contribution of the W to the fermion wave function renormalization.

3. the graphs $V_1, \cdots, V_4$ represent the contribution of the W to the radiative correction of the photon-fermion vertex.

4. the graph B is a box diagram, where a W^+ and a W^- are exchanged in the momentum transfer channel.

We shall use this scattering process in order to define the physical coupling constant g_{ph}. Calling $\mathcal{M}$ the scattering amplitude, we define g_{ph} by means of the residue at zero momentum transfer $q^2 = 0$:

$$g_{ph}^2 = i \lim_{q^2 \to 0} [q^2 \mathcal{M}] \ \cdot 4 \frac{m_1 m_2}{(p_{1\mu} p_2^\mu)} \quad , \tag{7}$$

where $p_{1,2}$ are the four momenta of the incoming particles.

Now, the important point to notice is that of all the diagrams of Fig1, the only one which contributes to the pole is P_1. This is so because at $q^2 \to 0$ the pole part of the diagrams $V_1, \cdots, V_4$ cancels against the contribution of the diagrams $E_1, \cdots, E_4$. This is a consequence of the $U(1)$ Ward identity, which we have explicitly checked to hold on the diagrams involving the W. As for the box diagram B it does not have any pole for $q^2 \to 0$. Concerning the diagram P_2, it corresponds to a double pole, which, by gauge invarince, cancels against the double pole part of P_1.

The sum of the diagrams P_1 and P_2 gives the contribution:

$$P_1 + P_2 = J_1^\mu \left(-i \frac{g_{\mu\rho}}{q^2} \right) i \Pi^{\rho\sigma} \left(-i \frac{g_{\sigma\nu}}{q^2} \right) J_2^\nu \quad , \tag{8}$$

where

$$J_1^\mu = -i \frac{g}{2} \overline{u}_1(p_1 + q) \gamma^\mu u_1(p_1) \quad , \quad J_2^\nu = i \frac{g}{2} \overline{u}_2(p_2 - q) \gamma^\nu u_2(p_2) \quad ,$$

are the fermion currents, and the vacuum polarization tensor is:

$$\Pi_{\rho\sigma} = (-g_{\rho\sigma} q^2 + q_\rho q_\sigma) F(q^2) \ . \tag{9}$$

We thus have, using current conservation:

$$P_1 + P_2 = i \frac{F(q^2)}{q^2} J_{1\mu} J_2^\mu \ . \tag{10}$$

The Feynman graph P_1 corresponds to the following contribution to $\Pi_{\rho\sigma}$:

$$i \ \Pi_{\rho\sigma}^{(P_1)} = (i g \mu^\epsilon)^2 \int \frac{d^D k}{(2\pi)^D} V_{\rho\alpha\beta}(q, k, k+q) \ i D^{\alpha\delta}(k) \ i D^{\beta\gamma}(k+q) \ V_{\sigma\gamma\delta}(-q, k+q, k) \ . \tag{11}$$

Here

$$iD_{\mu\nu}(k) = i\frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \ . \tag{12}$$

is the W-propagator in the unitary gauge and

$$V_{\lambda\mu\nu}(k_1, k_2, k_3) = g_{\lambda\mu}(k_1 - k_2)_\nu + g_{\mu\nu}(k_2 + k_3)_\lambda - g_{\nu\lambda}(k_3 + k_1)_\mu \ . \tag{13}$$

We make the computation in the dimensional regularization scheme, where $D = 4 - 2\epsilon$ and $g \to g\mu^\epsilon$, with μ the regularization scale.

Upon adding the contribution to $\Pi_{\rho\sigma}$ from the graph P_2, which is q-independent and cancels a similar term from P_1, we get the $U(1)$ gauge invariant expression eq.9, with

$$F(q^2) = \frac{g^2}{16\pi^2}\left[-7 + \frac{7}{6}\frac{q^2}{M_W^2} + \frac{1}{12}\frac{q^4}{M_W^4}\right]\left(\frac{1}{\epsilon} - \log\frac{M_W^2}{4\pi\mu^2}\right) + \frac{g^2}{16\pi^2}G(q^2) \ . \tag{14}$$

$G(q^2)$ is finite in the limit $\epsilon \to 0$ with value at $q^2 = 0$:

$$G(0) = \frac{2}{3} + 7\gamma \ .$$

γ being the Euler constant.

The above result eq.14 was already available in the literature [8] and we have checked it by an independent computation.

We point out that the expression for F contains non-renormalizable divergent terms, namely those of order q^2/M_W^2 and q^4/M_W^4. The presence of such terms in the expression of the Green's functions is a peculiarity of the unitary gauge and it is in this sense that it is usually referred to as a non renormalizable gauge. These terms originate from the bad ultraviolet behavior of the massive vector boson propagator (eq.12). Those nonrenormalizable divergences cancel from the on-shell scattering amplitude [6, 4, 5]. In fact, other graphs of Fig.1 have also divergences higher than usual. The vertex parts of the diagrams $V_1, \cdots, V_4$ have divergent terms proportional to q^2/M_W^2 and q^4/M_W^4 (to be precise, V_1 and V_3 turn out to be finite), while the box diagram B has a divergent term proportional to $1/M_W^2$ and another proportional to q^2/M_W^2. By keeping into account the $1/q^2$ of the photon propagator and adapting to our case the available expressions for those divergent terms [8], one can check that the nonrenormalizable divergences cancel from the amplitude.

Thus, the only divergent term that remains in the amplitude is the one occurring in $F(q^2 = 0)$ which is removed by the photon wave function renormalization constant Z, as it happens in ordinary QED. Indeed eq.7 gives to one-loop order:

$$i \lim_{q^2 \to 0} [q^2 \mathcal{M}] \ \cdot 4 \frac{m_1 m_2}{(p_{1\mu} p_2^\mu)} \ = g^2 (Z^{-1} - F(0) - F_{QED}(0)) \ , \tag{15}$$

where $F_{QED}(q^2)$ is the standard contribution of the fermions in spinorial QED. This allows us to immediately read off this renormalization constant in the minimal subtraction scheme:

$$Z = 1 + 7 \frac{g^2}{16\pi^2} \frac{1}{\epsilon} + \delta Z_{QED} \ , \tag{16}$$

where δZ_{QED}, the contribution of the two families of fermions doublets, reads:

$$\delta Z_{QED} = -\frac{4}{3} \frac{g^2}{16\pi^2} \frac{1}{\epsilon} \ . \tag{17}$$

By defining the renormalized coupling constant in terms of the bare one g_0 as

$$g = g_0 \mu^{-\epsilon} Z^{1/2} \tag{18}$$

(we have made use of the QED Ward identity) we compute the β function

$$\beta(g) = -\frac{g^3}{16\pi^2} \left(7 - \frac{4}{3} \right) \ . \tag{19}$$

The same result is obtained for the unbroken $SU(2)$ gauge theory, with a scalar multiplet in the adjoint representation and two fermion families in the fundamental representation. The number 7 in eq.19, which represents the massive vector contribution precisely accounts for the combined contribution of the $SU(2)$ massless gauge fields and Higgs scalars of the unbroken phase: $-7 = -22/3 + 1/3$. It is amusing to note that while in the unbroken $SU(2)$ theory one gets the β-function from the computation of several diagrams, including in general both vertex and self-energy parts, in the $U(1)$ phase the same result comes just from the evaluation of the photon self-energy.

It may be useful to stress that the term $k_\mu k_\nu / M_W^2$ in the propagator eq.12, while is responsible of the nonrenormalizable divergences in eq.14, is also essential to obtain the correct result for $q^2 \to 0$.

We may also further remark that it is the lowest, i.e. renormalizable, divergence which contributes the negative term $-7/\epsilon$ in $F(q^2)$ eq.14, while the highest non-renormalizable one has a positive sign. In usual QED, without charged massive vectors, the highest divergence in $F_{QED}(q^2)$ is the renormalizable one and it has a positive sign, giving a term in the β-function which is opposite to UV asymptotic freedom.

Upon inserting eq.14 and the known expression for $F_{QED}(q^2)$ into eq.15 we get:

$$g_{ph}^2 = g^2\left(1 - \frac{g^2}{16\pi^2}\left[7\log\frac{M_W^2}{\tilde{\mu}^2} + \frac{2}{3} - \frac{2}{3}\log\frac{m_1^2}{\tilde{\mu}^2} - \frac{2}{3}\log\frac{m_2^2}{\tilde{\mu}^2}\right]\right) \ . \tag{20}$$

where $\tilde{\mu}^2 = 4\pi e^{-\gamma}\mu^2$. Here g^2 has to be considered as a reference value, independent of the masses of the particles. By summing over the iterations of the self-energy corrections, eq.20 can be recast in the following convenient form

$$\frac{1}{g_{ph}^2} = \frac{1}{g^2} + \frac{1}{16\pi^2}\left[7\log\frac{M_W^2}{\tilde{\mu}^2} + \frac{2}{3} - \frac{2}{3}\log\frac{m_1^2}{\tilde{\mu}^2} - \frac{2}{3}\log\frac{m_2^2}{\tilde{\mu}^2}\right] \ . \tag{21}$$

This equation exhibits the mass dependence, characteristic of the effective coupling of a broken non-abelian theory, see for instance [9].

As a final remark, in the $N = 2$ supersymmetric $SU(2)$ Y.M. theory, broken to $U(1)$, one has to evaluate the r.h.s. of eq.21 by taking into account, in the place of our fermion families, the supersymmetric partners of the W. One thus gets [3]:

$$\frac{1}{g_{ph}^2} = \frac{1}{g^2} + \frac{1}{4\pi^2}\log\frac{M_W^2}{\tilde{\mu}^2} \ . \tag{22}$$

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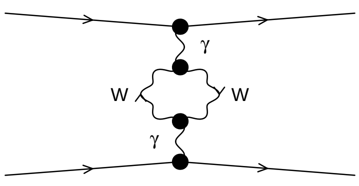

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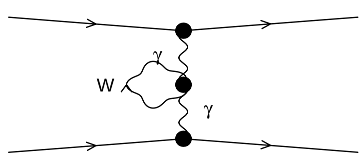

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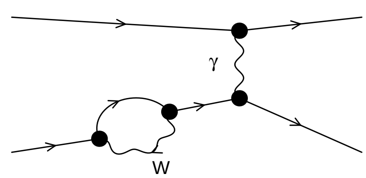

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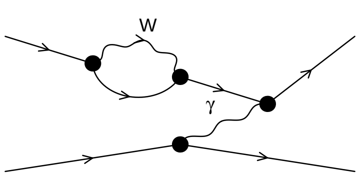

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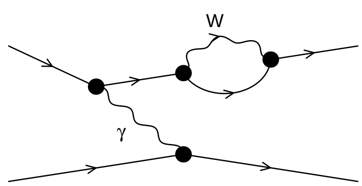

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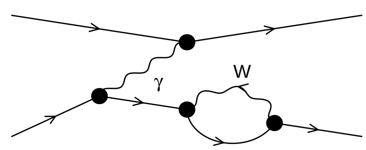

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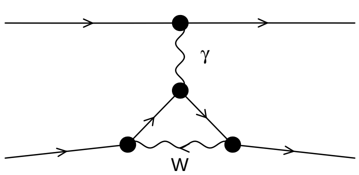

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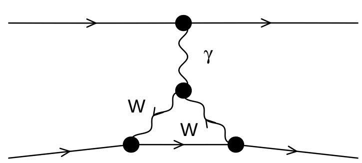

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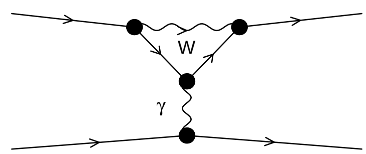

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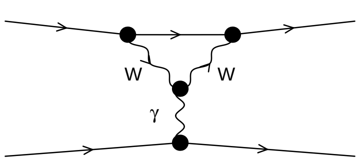

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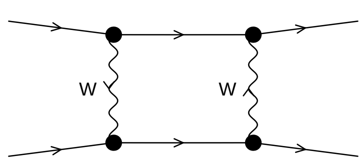

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